- **Square Wave**
  - $S(x) = A(-1)^{\left(\frac{2(x-a)}{T}\right)}$ A = amplitude, T = period, a = time lag
  - $S(x) = A\operatorname{sgn}\left(\sin\left(\frac{2\pi(x-a)}{T}\right)\right)$
  - Fourier series: $F(x) = \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left(\frac{2\pi(2k-1)(x-a)}{T}\right)$
- **Pulse Wave**
  - $f(x) = A\sum_{n=-\infty}^{\infty}\Pi_{nT,nT+\tau}(x-a)$
  - A = height, T = period, a = time lag, $\tau =$ duration of pulse, assume $\tau < T$.
  - Fourier series: $F(x) = \frac{A\tau}{T} + \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi\tau}{T}\right)\cos\left(\frac{2n\pi x}{T}\right)$
- **Triangle Wave**
  - $f(x) = \frac{2A}{\pi}\sin^{-1}\left(\sin\left(\frac{2\pi(x-a)}{T}\right)\right)$ A = amplitude, T = period, a = time lag
  - Fourier series: $F(x) = -\frac{8A}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)^2}\sin\left(\frac{2\pi(2n-1)(x-a)}{T}\right)$
- **Sawtooth Wave**
  - $f(x) = -\frac{2A}{\pi}\tan^{-1}\left(\cot\left(\frac{\pi(x-a)}{T}\right)\right)$ A = amplitude, T = period, a = time lag
  - Fourier series: $F(x) = -\frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi(x-a)}{T}\right)$

- These functions can all be represented using Fourier series, which uses a series of sines and cosines to approximate these discontinuities.
- How to solve a linear non-homogeneous differential equation with a discontinuous forcing term $f(x)$:
  - Method 1. Fourier series
    - Represent $f(x)$ using a Fourier series.
    - Use the equation $y_p = \frac{e^{\alpha x}}{p(\alpha)}$ with superposition.
  - Method 2. Laplace transform (more on next page)
    - $\mathcal{L}[f(t)] = \frac{1}{1-e^{-Ts}}\int_0^T f(t)e^{-st}dt$ given $f(t)$ is periodic with period *T*.